The idea that tasks play a pivotal role in a mathematics lesson has a long standing in mathematics education research. Recent calls for teaching reform appeal for training teachers to better understand how students learn mathematics and to employ students’ mathematical thinking as the basis for pedagogy, planning, and instructional decisions (CCSSM, 2010; NCTM, 2000; NRC 1999). The coupling of the teaching practices of (a) developing a task for a mathematics lesson and, (b) modifying the task for students while enacting the lesson fits within the scope of supporting students’ mathematical thinking.

Surprisingly, an extensive search of the literature did not yield any investigations by researchers who aimed to identify and refine the constituent parts of the aforementioned teaching practices in the manner called for by Grossman and colleagues (2009). The literature did yield research that focused on the characteristics of mathematics problems (e.g., Land & Drake, 2014), but these studies seemed limited in the sense that the teaching practice of designing a problem was not explored from the perspective of expert mathematics teachers.

Consequently, my research addresses the two questions: (a) what factors do exemplary elementary teachers consider when developing a task for a mathematics lesson? (b) what factors do they consider when they modify a task for a student when enacting a lesson?

To answer these questions, I conducted a multiple case study involving three elementary teachers, each with extensive training in the area of Cognitively Guided Instruction (CGI), as well as several years experience teaching mathematics following the principles of CGI (Carpenter et. al., 1999). I recorded video of three mathematics lessons with each participant, and within a few hours after each lesson I conducted a semi-structured stimulated recall interview with the respective teacher. A subsequent follow-up clinical interview was conducted soon thereafter to further explore the teacher’s thoughts regarding the problem and the lesson (Ginsberg, 1997). In addition, my methodology included interjecting myself at select times during a lesson to ask a teacher to explain her reasoning immediately after she modified a problem.

Qualitative analysis of the data led to a framework that identified four categories of influencing factors and seven categories of supporting objectives for the development of a task during the design phase of a lesson. Consequently, subsets of these factors and objectives emerged as particularly relevant when the teachers decided to modify a task during the enactment phase of a lesson. Moreover, relationships between and among the various factors were identified. The emergent framework from this study offers insight into decompositions of the two teaching practices of interest, in particular the utility of the number choices made by the teachers.